



A comprehensive fracture network conductivity model for tight unconventional reservoirs considering various proppant size, creep deformation, and proppant compaction and embedment

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ABSTRACT

Graded proppant injection into complex fractures is frequently used to prop connected secondary fractures in tight unconventional reservoirs. A comprehensive conductivity model incorporating creep, decreasing proppant size distribution, proppant embedment and deformation, and unproped fracture surface deformation is established to ascertain partially propped fracture network conductivity. The propped fracture width variation is described by creep deformation, proppant embedment, and proppant particle deformation. The corresponding fracture permeability is depicted by the Carman-Kozeny equation where the dynamic proppant pack porosity is calculated via proppant size and the number of proppant layers. For unproped areas, the width is controlled by effective stress, and the permeability is a function of fracture aperture. The hydraulic-electric analogies concept is used to integrate the local conductivity of different areas and characterize the overall fracture network conductivity. The model is verified against long-term conductivity measurement data. Results show that the fracture width variation is mainly caused by rock creep and proppant embedment. Larger Kelvin shear modulus and Maxwell viscosity slow down the conductivity decline rate. The conductivity becomes stable after 4 days when the Kelvin shear modulus is increased to 5.4×10^8 Pa. The Maxwell shear modulus has the slightest influence on conductivity. Larger-size proppants offer higher overall conductivity and better maintain the conductivity. The fracture network conductivity is significantly larger than the conductivity of the main fracture fully supported by the graded proppants and that of the fracture branches. The three-dimensional (3D) conductivity diagram and two-dimensional (2D) conductivity maps are generated to better demonstrate time-dependent conductivity evolution.

1. Introduction

Tight unconventional reservoirs, such as shale, coal, and tight sand formations, normally require massive hydraulic fracturing technology to recover oil and gas resources at economical rates [1,2]. During hydraulic fracturing, the proppant-laden fluid is injected into the reservoirs to generate high-conductivity hydraulic fractures and open natural

fractures [3]. After reservoir stimulation, the created artificial fractures and the connected natural fractures serve as the major effective flow channels for hydrocarbon extraction [4]. Most proppants are placed within the main hydraulic fractures to offer certain conductivity and avoid complete fracture closure. Therefore, the permeability or conductivity of different sizes of proppant-filled fractures plays a key role in maintaining the long-term well productivity. The conductivity is defined

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as fracture permeability times width [5]. The uniformity of proppant distribution within these fractures is a critical factor determining the overall fracture permeability/conductivity [6]. Besides, for different types of tight unconventional reservoirs, the fracture geometry varies substantially from single bi-wing fractures to fracture networks, as shown in Fig. 1(a). Mixed-sized proppant injection technology is proposed to realize rational proppant placement and better maintain conductivity of fracture tips or branches, as shown in Fig. 1(b) [7,8]. The main influencing factors for fracture conductivity include proppant properties (particle size, number of proppant layers, Young's modulus, and Poisson's ratio), stress state, rock mechanical properties, proppant damage mechanisms, rock hydration expansion, etc [9]. Therefore, it is challenging to characterize the overall conductivity of these fractures and accurately predict conductivity evolution during the whole production period.

In the literature, a lot of research has been conducted trying to figure out appropriate fracture conductivity models. Some typical ones are summarized in Table 1. Schmidt et al. [11] performed an investigation on the fracture conductivity offered by mixed-sized proppants. The well-blended or mixed proppants are described via parallel beds with linear flow. Thus, the overall conductivity is averaged by the weighted sum of each bed. They found that 40/80 mesh lightweight ceramic proppants provide sufficiently higher conductivity compared with 40/70 mesh sand. Larger-size lightweight ceramic proppants mixed with 40/70 mesh sand enhance overall proppant pack conductivity. Fan et al. [8] studied the conductivity of mixed proppants through the Lattice Boltzmann (LB) simulation. Their results showed that the conductivity first increases to a peak value with proppant concentration increasing. After the peak value, the conductivity drops and then rises with the further increment of proppant concentration. Li et al. [12] derived a fracture conductivity model integrating proppant embedment, proppant deformation, and dynamic fracture aperture for single- or multi-layer proppant placement patterns. Their model can be used to select proppants with appropriate properties. Zhang et al. [13] proposed a conductivity model considering the proppant size, sorting, crushing, packing, rearrangement of particles, embedment, and superficially, water-induced conductivity damage. They used empirical coefficients to characterize the factors related to water damage, and found that clay content is the dominant factor for water damage. Based on the experimental analysis of proppant pack conductivity damage, Awolake et al. [14] proposed an empirical permeability model suitable for short-term fracture conductivity prediction in tight sandstone reservoirs. Wu and Sharma [15] developed a numerical model describing stress-driven fracture closure and the corresponding fracture conductivity. Their model is capable of handling heterogeneous surface, elastoplastic deformation, and deformation interaction. They noticed that the multiple asperity interaction should not be ignored, and the conventional elastic-only model tends to overestimate fracture conductivity. Shamsi et al. [2] combined the discrete element method (DEM) and the LB model to study the dynamic conductivity of proppant-filled fractures under various stress conditions. Their results show that well-graded

proppant pack can form a strong bond during shearing via interlocking proppant grains. The uniformly-graded proppant pack has higher conductivity but is unstable when grain sliding occurs. Similarly, Fan et al. [16] also combined the DEM and the LB simulation to analyze the role of proppant embedment in fracture conductivity. Their results showed that the magnitude of rock Young's modulus has a dominant effect on proppant embedment. Zhang et al. [17] coupled the DEM and computational fluid dynamics (CFD) and proposed an integrated work flow to simulate proppant embedment and fracture conductivity. They found that proppant embedment is mainly caused by the rock-hydration effect. Zhu et al. [18] presented a fracture conductivity model for channel-fracturing-treated formations with consideration of combined influences of proppant embedment and fracture-surface and pillar deformation. Their study revealed that there exists an optimal ratio of pillar diameter to its distance (about 0.6–0.7) for maximal fracture conductivity. Teng et al. [19] derived a new permeability model that can describe the viscous-shear-dominated regime, transition regime, and the Darcy-flow-dominated regime. Their research bridges the gap between the viscous-shear-dominated fracture permeability equation and the Darcy-flow-dominated permeability equation based on constant proppant pack permeability and porosity assumptions. Li et al. [20] presented a fracture conductivity calculation formula which is a function of proppant concentration, the maximum conductivity factor, net stress, the decay rate exponent, and the proppant concentration power exponent. Their simulation results indicate that creep deformation is a dominant factor for estimated ultimate recovery (EUR) prediction. Geng et al. [21] established a permeability model for proppant-filled smooth fractures based on 150 permeability numerical simulation values that are divided into training and validation groups. They analyzed the impacts of fracture aperture, porosity, and proppant size on fracture permeability. Wang and Cheng [4] further considered fracture tortuosity and branches and proposed a two-dimensional (2D) fracture permeability model based on the fractal theory. Their work indicates that fracture aperture distribution, fracture tortuosity dimension, and the fracture length ratio are influencing factors for fracture network permeability, while the fracture branch angle's impact is not significant. Liu et al. [10] built an analytical fracture conductivity model that addresses the impacts of multiple-size proppant, creep deformation, as well as proppant embedment. Their investigation on time-dependent and closure-pressure-dependent deformation helps to find the critical closure pressure, critical creep time, and the optimal proppant combination ratio for two-particle-size proppants. Cheng et al. [22] extended the model of Liu et al. [10] and proposed a comprehensive fracture conductivity model that integrates the impacts of multi-size proppant creep, embedment, and non-uniform propped fracture width distribution on fracture conductivity. To minimize the cumulative effect of creep on productivity, they suggested that the determination of the proppant size, type, mixing pattern, and operation parameters should consider rock mineralogy and mechanical properties. Zhang et al. [9] developed a propped fracture conductivity model incorporating compression-induced proppant damage, crushing, shale hydration, and

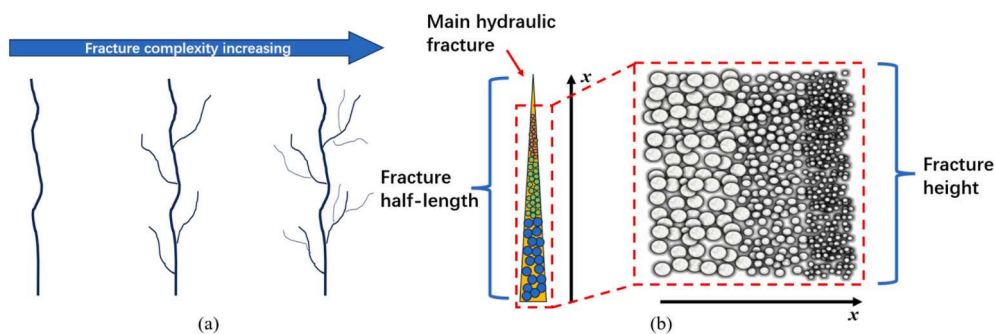


Fig. 1. Typical fracture geometry and proppant size distribution: (a) different fracture geometry and (b) mixed-sized proppant distribution in the main fracture [10].

Table 1
Typical fracture conductivity/permeability models.

Models	Model description	Key findings
Schmidt et al. (2014) [11]	Fracture conductivity model using weighted sum of each bed's conductivity for mixed-sized proppants	Larger-size lightweight ceramic proppants mixed with 40/70 mesh sand enhance overall proppant pack conductivity
Li et al. (2015) [12]	Fracture conductivity model integrating proppant embedment, proppant deformation, and dynamic fracture aperture with single- or multi-layer proppant placement patterns	The model can be used to select proppants with appropriate properties
Zhang et al. (2016) [13]	Fracture conductivity model considering the proppant size, sorting, crushing, packing, rearrangement of particles, embedment, and water-induced damage related to empirical coefficients	The clay content is the dominant factor for water damage
Awoleke et al. (2016) [14]	Empirical permeability model suitable for short-term fracture conductivity prediction in tight sandstone reservoirs	The applicability, modified form, and limitations of this model are discussed
Wu and Sharma (2017) [15]	Numerical conductivity model describing stress-driven fracture closure and considering heterogeneous surface, elastoplastic deformation, and deformation interaction	The conventional elastic-only model tends to overestimate fracture conductivity, and multiple asperity interaction should not be ignored
Shamsi et al. (2017) [2]	Fracture conductivity simulation combining the DEM and the LB model under various stress conditions	The uniformly-graded proppant pack has higher conductivity than the well-graded proppant pack but is unstable during grain sliding
Zhang et al. (2017) [17]	Fracture conductivity workflow coupling the DEM and CFD and considering proppant embedment	Proppant embedment is mainly caused by the shale-hydration effect
Zhu et al. (2019) [18]	Fracture conductivity model for channel-fracturing considering combined influences of proppant embedment and fracture-surface and pillar deformation	There is an optimal ratio of pillar diameter to its distance for maximizing fracture conductivity regardless of the proppant size
Fan et al. (2020) [8]	Characterizing conductivity of fractures with mixed proppants through LB simulation	Conductivity first increases to a peak value, then drops, and finally rises with proppant concentration increasing
Teng et al. (2020) [19]	Permeability model suitable for the viscous-shear-dominated flow, transition flow, and the Darcy flow	Bridging the gap between the viscous-shear-dominated fracture permeability equation and the Darcy-flow-dominated permeability equation
Wang and Cheng (2020) [4]	Fractal-theory-based 2D permeability model considering fracture tortuosity and branches	Fracture aperture distribution, tortuosity dimension, and the length ratio are influencing factors for fracture network permeability
Fan et al. (2021) [16]	Fracture conductivity model combining the DEM and LB simulation	Rock Young's modulus has a dominant effect on proppant embedment
Liu et al. (2022a) [23]	Analytical fracture conductivity model for rod proppants	Optimal rod proppant particle spacing in both radial and axial flow directions are obtained
Liu et al. (2022b) [10]	Analytical fracture conductivity model considering multiple-size proppant, creep deformation, and proppant embedment	Critical closure pressure, critical creep time, and the optimal proppant combination ratio for two-particle-size proppants are obtained
Li et al. (2023) [20]	Fracture conductivity model as a function of proppant concentration, maximum conductivity factor, net stress, decay rate exponent, and the	Creep deformation is a dominant factor for accurate EUR prediction

Table 1 (continued)

Models	Model description	Key findings
	proppant concentration power exponent	
Geng et al. (2023) [21]	Proppant-filled permeability model for smooth fractures based on 150 permeability numerical simulation values	Impacts of fracture aperture, porosity, and proppant size on fracture permeability are obtained
Cheng et al. (2023) [22]	Fracture conductivity model integrating multi-size proppant creep, embedment, and non-uniform fracture width distribution	Suggestions for determination of proppant size, type, mixing pattern, and operation parameters are obtained
Wang et al. (2024) [24]	Fracture conductivity calculation model considering the proppant placement pattern, embedment, and proppant deformation	Conductivity decreasing involves three stages: the embedding stage, the creep stage, and the stabilization stage
Zhang et al. (2025) [9]	Fracture conductivity model involving proppant damage, crushing, shale hydration, and embedment	The rank of influencing factors is: proppant size > proppant Young's modulus > fracture fluid pressure > hydration-induced expansion > filtration depth

embedment. Sensitivity investigation indicates that the rank of influencing factors is: proppant size > proppant Young's modulus > fracture fluid pressure > hydration-induced expansion > filtration depth. Unlike most researchers focusing on the conductivity provided by spherical proppants, Liu et al. [23] derived an analytical model for fracture conductivity offered by rod proppants used in the pulse-fracturing process to avoid flowback. They obtained optimal rod proppant particle spacing in both radial and axial flow directions. Wang et al. [24] simultaneously considered the proppant placement pattern, proppant embedment, and proppant deformation when calculating complex fracture's long-term conductivity. They concluded that the time-dependent conductivity decreasing can be divided into three stages: embedding, creep, and stabilization stages.

Although the above fracture conductivity or permeability models are versatile enough to replicate and explain conductivity evolution behavior under certain testing conditions, existing research on fracture conductivity seldom simultaneously considers multiple-size proppants, proppant embedment and deformation, rock creep, fracture branches, and the overall conductivity of fracture networks. Some researchers integrated multi-size proppants, creep, proppant embedment on their conductivity models, the ways they calculate the relation between proppant pack dynamic porosity and permeability are simply using the cubic law or simplified proppant pack porosity characterization [10,22]. In this study, a generic conductivity model for fracture networks is established incorporating the impacts of proppant deformation, embedment, multi-sized proppant placement, and creep deformation in propped fracture segments, and deformation of unpropped fracture branches on fracture conductivity. To accurately describe the dynamic proppant pack properties, the proppant pack porosity is calculated based on the stress state and the proppant pack apparent mechanical properties. Then, the proppant pack dynamic permeability is expressed a function of its porosity via the Carman-Kozeny equation. Specifically, the hydraulic-electric analogies are utilized to combine the conductivity of both propped fracture segments and unpropped fracture branches to depict fracture network conductivity. In this way, the above-mentioned limitations of previous models can be addressed.

2. Model development

The research aim of this study is to develop a generic fracture conductivity model for stimulated tight unconventional reservoirs. In this section, the conceptual model and corresponding assumptions are introduced first. Then, the mathematical model and the its validation are

presented.

2.1. Conceptual model

The proppants are spherical particles. To address multi-size proppant injection [10], the particle size can be different, as shown in Fig. 2(a). The proppant pack can be either single-layer or multi-layer. For multi-layer proppants, their accumulation pattern is shown in Fig. 2(b). Point O in Fig. 2(b) is the centroid of the triangle of which the three vertexes are the center points of the three spherical proppant particles in the lower layer. By following the assumption of Liu et al. [10], in the fracture length direction, proppants are arranged through decreasing sizes. The created fracture system can be a single bi-wing fracture or a fracture network with a propped main fracture and partially propped fracture branches. The hydraulic-electric analogies are applied to calculate the equivalent overall fracture network conductivity [25], as shown in Fig. 2(c). With the closure stress, proppant migration is not considered here. The fracture surface is smooth so that the fractures are planar fractures with possible planar fracture branches. The fracture width values at different fracture segment are not identical. Among the previously mentioned three types of unconventional reservoirs, this model is particularly suitable for shale and coal formations with complex and partially propped fracture networks. As for tight sand formations where bi-wing fractures are more likely to be created, degraded form of this model with fewer or no fracture branches can be used to calculate the conductivity of partially propped bi-wing fractures. The flow direction within the fracture network during production is from fracture tips to the fracture-wellbore interface (see Fig. 2(c)).

In reality, the proppant size is not identical. However, for a specific range of proppant size (a certain mesh range, for example, 20/40 mesh), one can use single-size proppants to equivalently calculate the conductivity of the proppant pack. This approximation has widely been verified by comparing calculation results with exact conductivity measurement data [10]. Another issue is that, graded proppant injection (for example, injecting 20/40 mesh, 40/70 mesh, and 70/140 mesh proppants in a certain order) may also lead to nonuniform distribution of proppant in the fracture segments. However, as can be seen in Fig. 3, both indoor experimental results and proppant distribution in field-scale hydraulic fractures show single-range-mesh-proppant segments [27]. This indicates that using a certain proppant size in a specific fracture segment

to describe graded proppant placement is applicable. Moreover, a certain range of mesh for a certain fracture segment is also in accordance with classical experimental design for conductivity measurement of fractures with graded proppant distribution, as shown in Fig. 4.

2.2. Mathematical model

2.2.1. Fracture width models

Fracture conductivity is defined as fracture permeability times width. Therefore, in this section, the fracture width model and the propped pack permeability model are presented respectively. As shown in Fig. 2(b), if the number of proppant layers is 2, the fracture width without proppant embedment is [26].

$$w_2 = h + 2r = \sqrt{(2r)^2 - \left(\frac{2\sqrt{3}}{3}r\right)^2} + 2r = \frac{2\sqrt{6}}{3}r + 2r \quad (1)$$

where h is the height of regular tetrahedron in Fig. 2(b)–m; r is the proppant radius, m; w is the fracture width, m; and the subscript 2 denotes the number of proppant layers. For more general cases, the number of proppant layer in the fracture width direction is n_w . The corresponding fracture width is:

$$w_f = (n_w - 1) \frac{2\sqrt{6}}{3}r + 2r \quad (2)$$

where w_f is the fracture width, m. As shown in Fig. 5, the fracture segment length and height are expressed as [9]

$$l_f = 2rn_l \quad (3)$$

$$h_f = (n_h - 1)\sqrt{3}r + 2r \quad (4)$$

where l_f is the propped fracture segment length, m, and h_f is the propped fracture segment height, m. As a result, the particle numbers in the propped fracture length, height, and width directions are

$$n_l = l_f / (2r) \quad (5)$$

$$n_h = \sqrt{3}(h_f - 2r) / (3r) + 1 \quad (6)$$

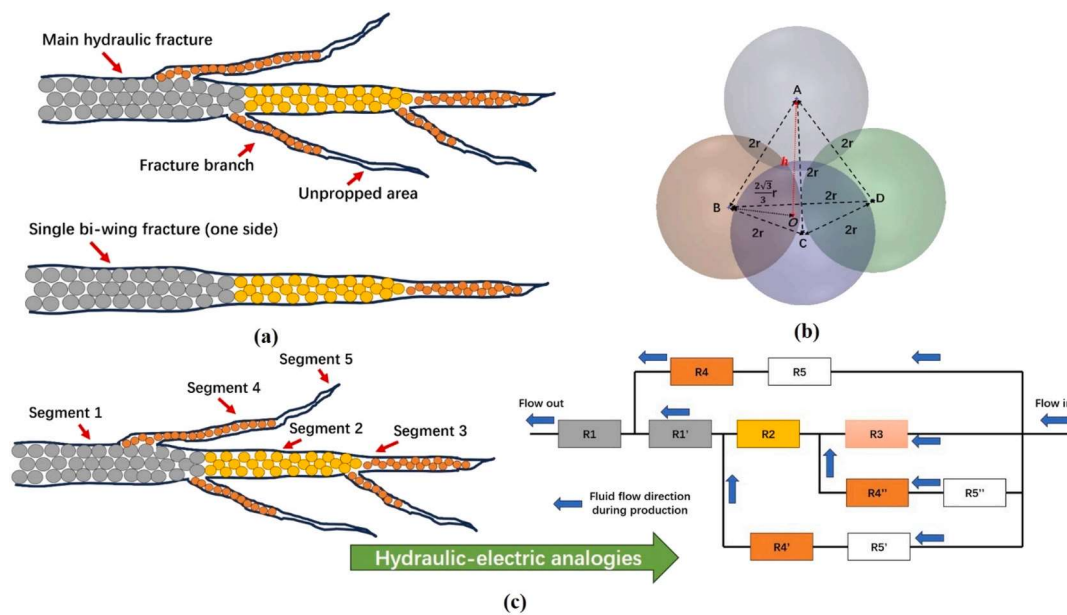


Fig. 2. Conceptual model: (a) schematic diagram of bi-wing and complex fractures with well-graded proppants, (b) accumulation pattern of proppants [26], and (c) schematic diagram of hydraulic-electric analogies.

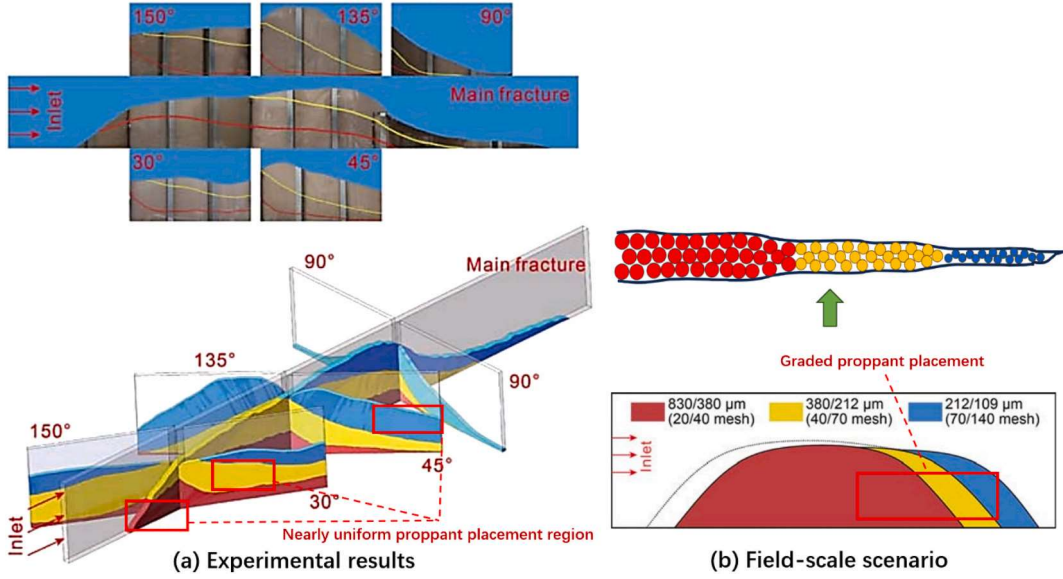


Fig. 3. Graded proppant distributions at the laboratory scale and the field scale (modified from Ref. [27]).

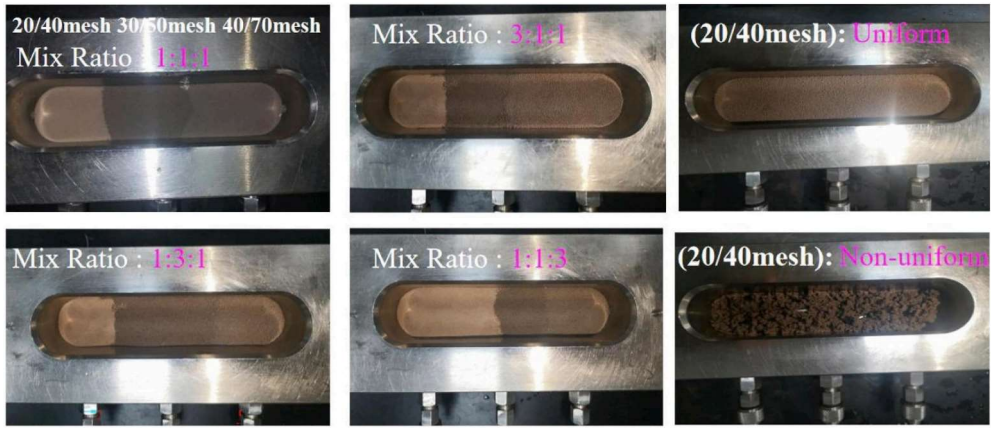


Fig. 4. Different fracture segments with different size of proppants [28].

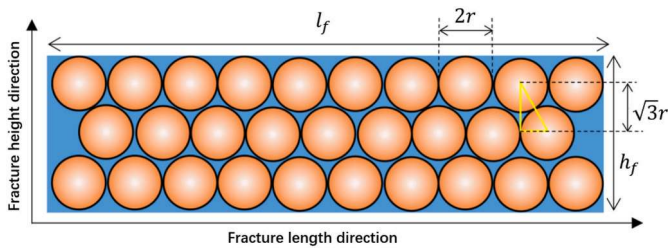


Fig. 5. Proppant distribution along the fracture height and length directions [9].

$$n_w = \sqrt{6}(w_f - 2r) / (4r) + 1 \quad (7)$$

Note that one should use the integer values of Eqs. (5)–(7). The total number of proppants for a fracture segment is [9].

$$n_{total} = \begin{cases} n_l n_h n_w, & (l - l_f = r) \\ n_l n_h n_w - n_h \text{floor}(n_w/2), & (0 < l - l_f < r) \end{cases} \quad (8)$$

where n_{total} is the total number of proppant particles within a fracture segment, and l is the fracture segment length, m. The floor function

(floor(x)) gives the largest integer less than or equal to x . The total number of proppants is used to calculate the solid particle volume within the segment. Thus, the porosity can be calculated.

Before calculating the change of fracture width and permeability, the force analysis should be conducted first, as shown in Fig. 6. It is the basis for rock and proppant deformation calculation. In Fig. 6, p_{ec} is the effective stress of the fracture surface, Pa; p_{ec1} is the resulting pressure generated by the action of the vertical closure pressure, Pa; d_e is the embedment depth, m; δ is the rock-proppant-compression-induced deformation, m; p_s is the stress between adjacent proppant particles, Pa; δ_{in} is the contact deformation between proppants, m; and a_e is the radius of the contact circle, m.

The radius of the contact circle is expressed as [30]:

$$a_e = \left(\frac{3p_{ec} l_{pr}^2 r}{4E^*} \right)^{1/3} = r \left(\frac{3p_{ec}}{E^*} \right)^{1/3} \quad (9)$$

where E^* is the effective elastic modulus, Pa; and l_{pr} is the distance between two proppant particle center points in the fracture length or height direction, and is not the distance between two particles in the fracture width direction (see Fig. 4(c)). The total force at the contact area between proppant and fracture surface is only in the direction perpendicular to the surface. Thus, $l_{pr} = 2r$. The embedment depth at

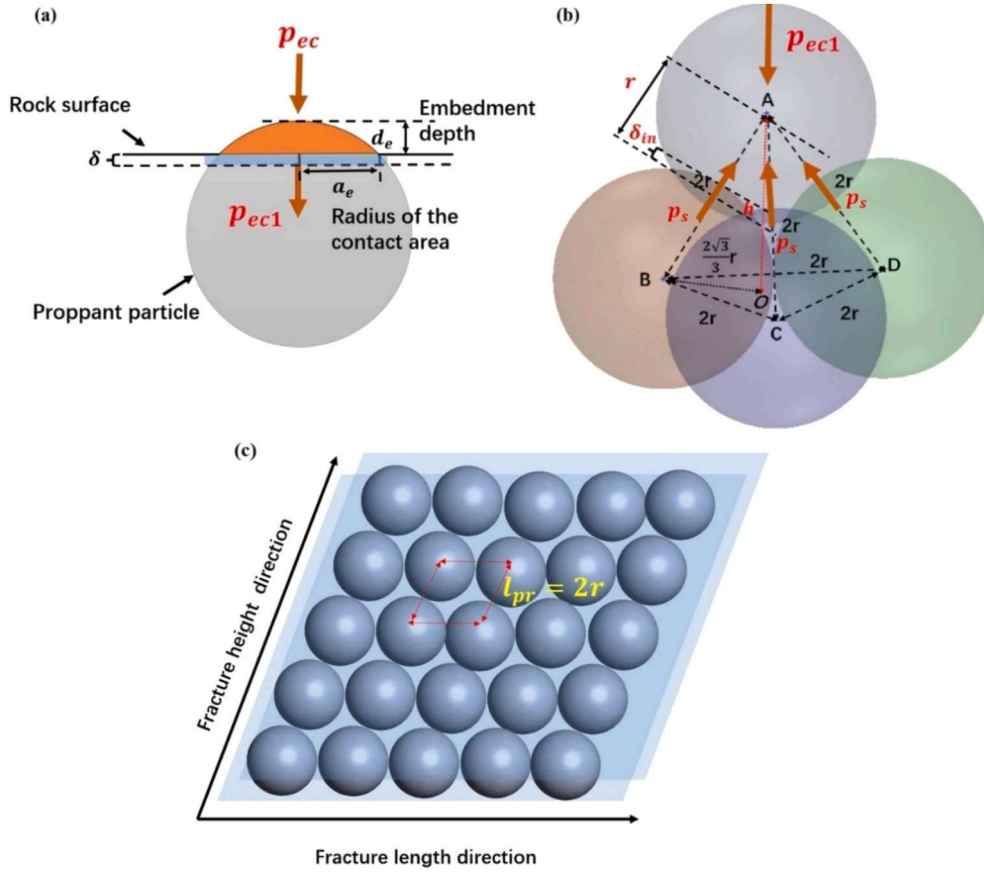


Fig. 6. Force analysis and particle deformation: (a) force analysis and deformation at the proppant-rock interface, (b) force analysis and deformation among proppant particles, and (c) the distance between two particles in the fracture length or height direction (modified from Refs. [26,29]).

the fracture surface is [30]

$$d_e = \left(\frac{3p_{ec}l_{pr}^2r}{4E^*} \right)^{2/3} \frac{1}{r} = r \left(\frac{3p_{ec}}{E^*} \right)^{2/3} \quad (10)$$

The fracture surface area is

$$A_{fs} = l_f h_f \quad (11)$$

The area calculated from the radius of the contact area is

$$A_{es} = \pi a_e^2 = \pi r^2 \left(\frac{3p_{ec}}{E^*} \right)^{2/3} \quad (12)$$

According to the force equilibrium relationship, one can obtain the following equation:

$$p_{ec}l_f h_f = p_{ec1}\pi r^2 \left(\frac{3p_{ec}}{E^*} \right)^{2/3} n_l n_h = p_{ec1}\pi r^2 \left(\frac{3p_{ec}}{E^*} \right)^{2/3} l_f / (2r)^* \left[\sqrt{3}(h_f - 2r) / (3r) + 1 \right] \quad (13)$$

Therefore, we have

$$p_{ec1} = \frac{p_{ec}l_f h_f}{\left(\frac{3p_{ec}}{E^*} \right)^{2/3} \pi r / 2^* \left[\sqrt{3}(h_f - 2r) / (3r) + 1 \right]} \quad (14)$$

Similarly, as shown in Fig. 6(b), the force on one proppant particle in

the vertical direction is

$$F_v = p_{ec1}\pi r^2 \left(\frac{3p_{ec}}{E^*} \right)^{2/3} \quad (15)$$

According to the force analysis, the force perpendicular to the contact area between proppant particles is

$$F_p = \frac{\sqrt{6}}{6} F_v = \frac{\sqrt{6}}{6} p_{ec1}\pi r^2 \left(\frac{3p_{ec}}{E^*} \right)^{2/3} = \frac{\sqrt{6}}{3} \frac{p_{ec}h_f r}{\left[\sqrt{3}(h_f - 2r) / (3r) + 1 \right]} \quad (16)$$

There is no need to calculate the actual stress between the proppants (p_s). One can use the force in Eq. (16) to obtain the deformation of proppants, δ_{in} [30]. The variation of the width of the propped fracture segment comes from three processes: proppant embedment at the fracture surface, proppant deformation, and creep deformation of the rock [22]:

$$w_{ft}(t) = w_{f0} - [2w_{em} + 2r\Delta\varepsilon_{fp} + 2H\Delta\varepsilon_c(t)] \quad (17)$$

where w_{em} is the sum of embedment depth (d_e) and contact deformation between proppants; ε_{fp} is the contact strain of the proppant caused by the fracture surface; H is the thickness of a shale plate, m; and ε_c is the rock strain induced by creep deformation. In Eq. (17), the initial fracture

width is

$$w_{f0} = (n_w - 1) \frac{2\sqrt{6}}{3} r + 2r \quad (18)$$

For the single-layer-proppant condition, there is no contact deformation between adjacent proppants. Only embedment depth at the fracture surface exists [30]:

$$w_{em} = d_e = \left(\frac{3p_{ec} l_{pr}^2 r}{4E_p^*} \right)^{2/3} \frac{1}{r} = r \left(\frac{3p_{ec}}{E_p^*} \right)^{2/3} \quad (19)$$

For multi-layer proppant placement conditions, both embedment depth and contact deformation between adjacent proppants exist. Thus, we have

$$w_{em} = d_e + (n_w - 1) \left[2r \cos 30^\circ - \sqrt{(2r - 2\delta_{in})^2 - r^2} \right] = r \left(\frac{3p_{ec}}{E_p^*} \right)^{2/3} + (n_w - 1) \left[2r \cos 30^\circ - \sqrt{(2r - 2\delta_{in})^2 - r^2} \right] \quad (20)$$

where [30].

$$\delta_{in} = \frac{\left(\frac{3F_p r}{4E_p^*} \right)^{2/3} \left\{ \frac{\sqrt{6} p_{ec} h_f r^2}{4E_p^* [\sqrt{3}(h_f - 2r)/(3r) + 1]} \right\}^{2/3}}{r} \quad (21)$$

In Eq. (21), E_p^* is the effective elastic modulus for proppant contact, Pa. For the proppant contact strain caused by the fracture surface, it is proportional to the stress [22]

$$\varepsilon_{fp} = \frac{p_{ec1}}{E_1} \quad (22)$$

The two-stage creep-induced rock strain is described by the Burgers model [22,31].

$$\varepsilon_c(t) = \frac{p_{ec}}{E_m} + \frac{p_{ec}}{E_k} \left[1 - \exp\left(\frac{-E_k}{\eta_k} t\right) \right] \quad (23)$$

where E_m represents the Maxwell shear modulus, Pa; E_k is the Kelvin shear modulus, Pa; t is time, s; and η_k is the Maxwell viscosity, Pa·s. Combining Eqs. (18)–(23), the dynamic propped fracture width can be obtained. As for the unpropped fracture segment, the stress-change-induced instantaneous elastic deformation is the major mechanism for fracture width and porosity variation. Thus, the change of fracture width is simplified as [32]:

$$\Delta w_{up} = -w_{up0} c_{fup} [(p_c - p_{c0}) - (p - p_0)] \quad (24)$$

where p_c is the closure pressure, Pa; p is the fluid pressure within the fracture, Pa; c_{fup} is the unpropped fracture compressibility; and w_{up0} is the unpropped fracture initial width.

2.2.2. Fracture permeability models

After obtaining the dynamic fracture width for different fracture segments, the next step is to figure out the corresponding permeability expressions. For the propped fracture segment, the permeability is a function of porosity according to the Carman-Kozeny equation. The initial porosity is related to the size and number of proppant particles [26]:

$$\phi_{pf0} = \frac{V_{p0}}{V_{f0}} = 1 - \frac{n_{total} \frac{4\pi r^3}{3}}{l h_f w_f} = 1 - \frac{4\pi r^3 n_{total}}{3 l h_f w_f} \quad (25)$$

where V_{p0} and V_{f0} are the initial fracture pore volume and initial propped fracture volume. According to the Carman-Kozeny equation, the corresponding permeability is given by [33]

$$k_{pf} = \frac{\phi_{pf}^3 d^2}{180(1 - \phi_{pf})^2} \quad (26)$$

Then, the evolution of permeability can be calculated via the change of propped fracture porosity. Liu et al. [34] used the apparent Young's modulus to describe the mechanical properties of the proppant pack. Here, we also use proppant pack bulk properties to calculate porosity evolution. Note that creep-induced proppant pack porosity change is negligible. The dynamic porosity is expressed as [35].

$$\phi_{pf} = \phi_{pf0} \exp\left\{ \left(\frac{1}{K} - \frac{1}{K_p} \right) [(p_c - p_{c0}) - (p - p_0)] \right\} \quad (27)$$

where K is the bulk modulus defined as $K = E_a / [3(1 - 2\nu_a)]$ [35,36], Pa, and K_p is the bulk modulus of the proppant pack pore space defined as $K_p = \phi_{pf0} K / \alpha$, Pa. α is the Biot coefficient defined as $\alpha = 1 - K/K_s$. K_s is the proppant particle bulk modulus expressed as $K_s = E_1 / [3(1 - 2\nu_1)]$. Here, E_a and ν_a are the apparent Young's modulus and Poisson's ratio of the proppant pack. Substituting Eq. (27) into Eq. (26) yields the dynamic fracture permeability of the propped fracture segment. For unpropped fracture segment, the permeability is a function of fracture width [10]

$$k_{upf} = \frac{w_{up}^2}{12} \quad (28)$$

2.2.3. Fracture conductivity model

As shown in Fig. 2, the whole fracture system is a combination of propped and unpropped fracture segments. Besides, in different propped fracture segments, the size of proppants is not identical. By assuming that fluid flow within the fractures obeys the Darcy's law, the flow rate of the i th fracture segment is [25].

$$Q_i = \frac{k_{fi} w_{fi} h_{fi} \Delta p}{\mu l_i} = \frac{\Delta p}{\left(\frac{\mu l_i}{C_{fi} h_{fi}} \right)}, i = 1, 2, 3, \dots, n \quad (29)$$

where the subscript i denotes the properties of the i th fracture segment; Q is the flow rate, cm^3/min ; Δp is the pressure difference between the inlet and outlet of the i th fracture segment, kPa; μ is the fluid viscosity, mPa·s; and C_f is the fracture conductivity, $\mu\text{m}^2 \cdot \text{cm}$. In a similar manner, the current can be expressed as

$$I = \frac{\Delta U}{R} \quad (30)$$

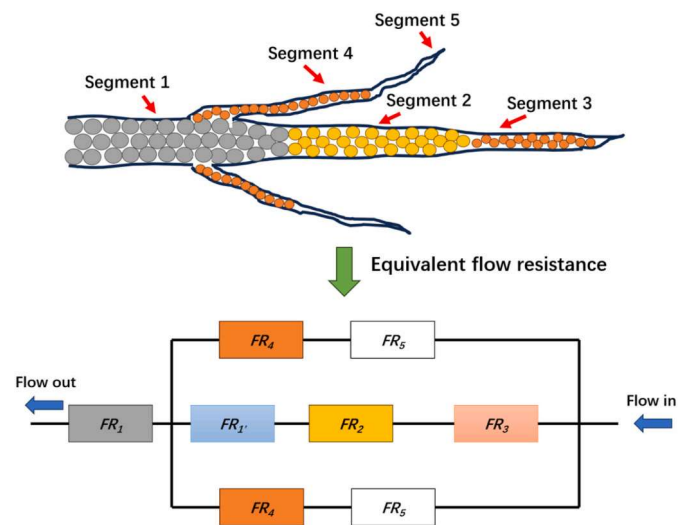


Fig. 7. Schematic diagram of a fracture network with different segments and the corresponding equivalent flow resistance.

where I is the current, A; ΔU the voltage difference, V; and R is the resistance, Ω . Comparing Eqs. (30) and (31), one can define the equivalent fluid resistance as

$$FR_i = \frac{\mu l_i}{C_{fi} h_{fi}} \quad (31)$$

Thus, as shown in Fig. 7, the total equivalent flow resistance can be expressed as

$$FR_t = FR_1 + \frac{1}{\frac{2}{FR_4 + FR_5} + \frac{1}{FR_{1'} + FR_2 + FR_3}} \quad (32)$$

Note that the fracture height is assumed to be identical for all the fracture segments. Consequently, the equivalent conductivity of the fracture network is

$$C_{ef} = \frac{\mu l_t}{FR_t h_f} \quad (33)$$

Here, l_t is the length of the main hydraulic fracture.

3. Model validation

Because the major novelty of the model is the new propped fracture conductivity model, model validation is performed by comparing conductivity measurement data of the propped fracture segment in shale samples. For the unpropped fracture conductivity model, it has already been frequently validated and used in the literature [37]. The shale plates used in the long-term conductivity measurement experiments were collected from the Vaca Muerta and Eagle Ford shale formations [38]. Brine was used as the flowing fluid at a constant pumping rate during 10 days. The closure stress was added through the axial load. The basic input parameters of the model for model validation are listed in Table 2. It should be noted that the values of these parameters are reasonable compared with those reported in the literature. Other

Table 2
Input parameters for model validation.

Basic Input Parameters			
Parameters	Values	Parameters	Values
Proppant radius (m) [22]	3.5×10^{-4}	Fracture length in the conductivity measurement cell (m) [38]	5.08×10^{-2}
Fracture height in the conductivity measurement cell (m) [38]	3.175×10^{-2}	One-side rock thickness (m) [38]	6.35×10^{-3}
Proppant particle Young's modulus (GPa) [22]	110	Proppant Poisson's ratio [22]	0.2
The Eagle Ford Shale Case			
Parameters	Values	Parameters	Values
Young's modulus (GPa) [38]	10.9	Poisson's ratio [22]	0.2
Apparent Young's modulus of the proppant pack (GPa) [34]	1.2	Apparent Poisson's ratio of the proppant pack	0.3
Maxwell shear modulus (GPa) [22]	4.135	Kelvin shear modulus (GPa)	0.2
Maxwell viscosity (GPa·s)	1.4×10^4	Closure stress (MPa) [38]	34.47
The Vaca Muerta Shale Case			
Parameters	Values	Parameters	Values
Young's modulus (GPa) [38]	20	Poisson's ratio [22]	0.2
Apparent Young's modulus of the proppant pack (GPa) [34]	6	Apparent Poisson's ratio of the proppant pack	0.3
Maxwell shear modulus (GPa) [22]	4.135	Kelvin shear modulus (GPa)	0.18
Maxwell viscosity (GPa·s)	7.3×10^4	Closure stress (MPa) [38]	34.47

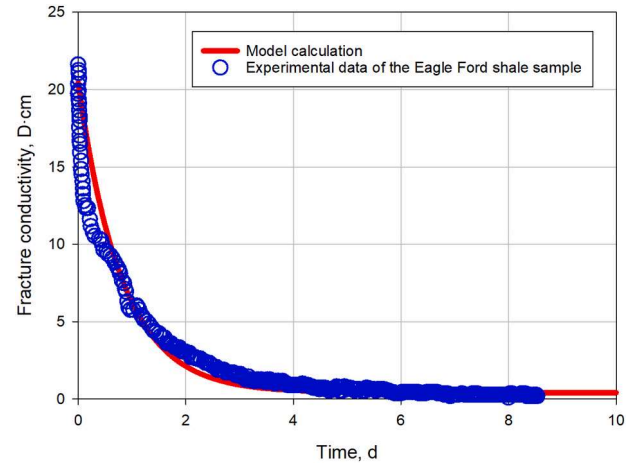


Fig. 8. Comparison between conductivity measurement data of the Eagle Ford shale sample [38] and model's calculation.

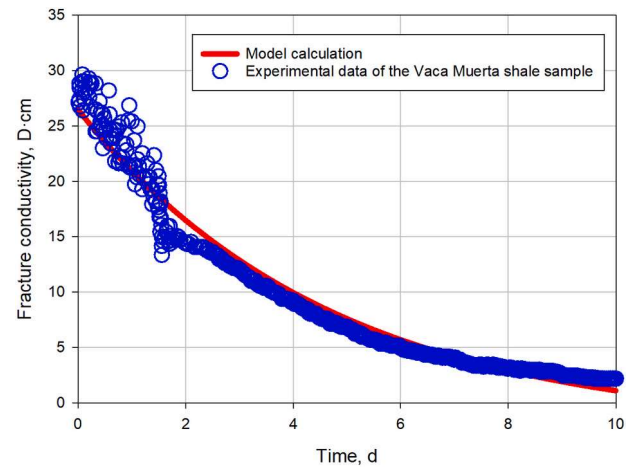


Fig. 9. Comparison between conductivity measurement data of the Vaca Muerta shale sample [38] and model's calculation.

parameters are obtained by experimental data matching, while their values are also reasonable for shale rocks. The differences in mechanical properties of the two samples come from their different rock mineral components [38]. The Eagle Ford shale sample's clay content is three

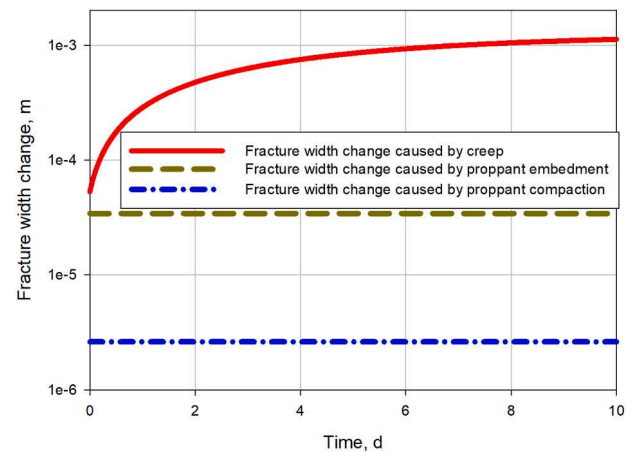


Fig. 10. Comparison of fracture width changes (one-side) caused by different deformation mechanisms.

times that of the Vaca Muerta shale sample. The fractures were filled with the 20/40 mesh proppants (Ottawa sand). For the two cases, proppants were placed in multiple layers. The equivalent number of layers may not be an integer after particle slippage during compaction [22]. The actual number of layers in the fracture width direction is determined by matching with the initial permeability (120 D). Figs. 8 and 9 show the comparison of model calculated and experimentally observed conductivity. In general, good agreements have been achieved between model's calculation and experimental observations of the two samples. Within the first 2 days, both samples experienced a dramatical conductivity loss (86 % reduction for the Eagle Ford shale sample and 46 % reduction for the Vaca Muerta shale sample). This phenomenon further indicates the heavy influence of creep deformation on fracture conductivity. The fracture width variation caused by different mechanisms is illustrated in Fig. 10. Proppant compaction offers the smallest contribution to fracture width variation. Initially, the impacts of proppant embedment and creep deformation are close to each other. Creep deformation induces the largest fracture width variation, especially at late times.

4. Results and discussion

After model validation, the impacts of different effective stress conditions and different fracture and proppant properties, such as the Maxwell viscosity for creep, unpropped fracture compressibility, and proppant size on different types of fracture segment conductivity, are analyzed. Then, the controlling factors of the overall fracture network conductivity are investigated.

4.1. Impacts of the creep-deformation-related rock mechanical properties

Creep-deformation-related mechanical parameters refer to the Maxwell viscosity (η_k), the Maxwell shear modulus (E_m), and the Kelvin shear modulus (E_k) in the Burgers model (Eq. (23)). As the Maxwell viscosity increases from 7×10^{13} Pa to 25×10^{13} Pa, the conductivity's decline with time slows down, as shown in Fig. 11. The conductivity curves' nonlinear relationship with time is gradually transformed into a more linear relationship. The initial conductivity is not changed. The conductivity differences become larger with time. As for the Kelvin shear modulus, a larger Kelvin shear modulus results in more stable late-time conductivity, as shown in Fig. 12. This indicates that, rocks with a relatively large Kelvin shear modulus experience a rapid conductivity drop in the first three days. After that time, the conductivity becomes relatively stable with a slow decline rate. As illustrated in Fig. 13, the change of the Maxwell shear modulus has no noticeable influence on fracture conductivity. The increase of the Maxwell shear modulus

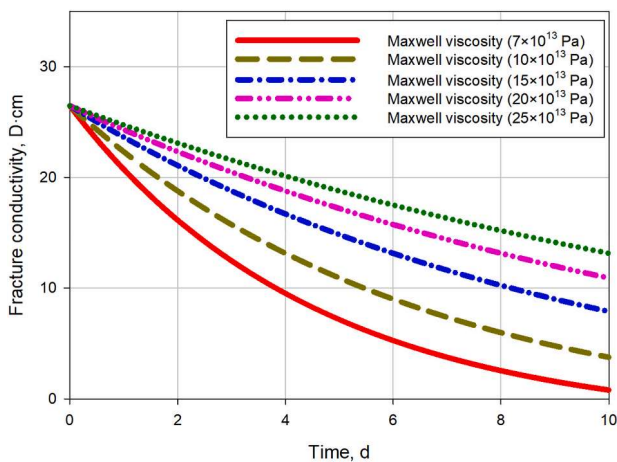


Fig. 11. Impacts of the Maxwell viscosity on fracture conductivity.

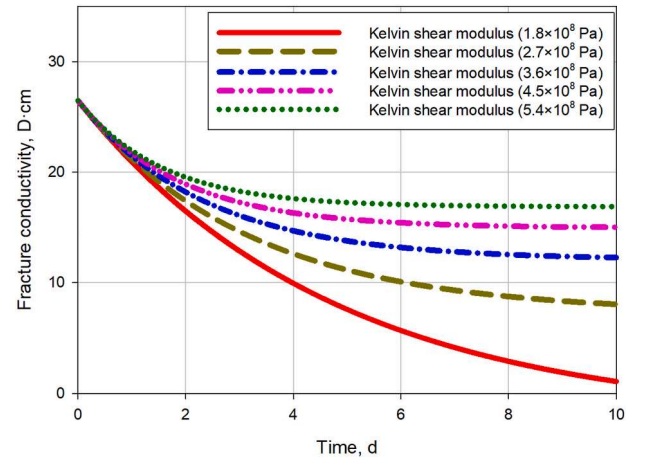


Fig. 12. Impacts of the Kelvin shear modulus on fracture conductivity.

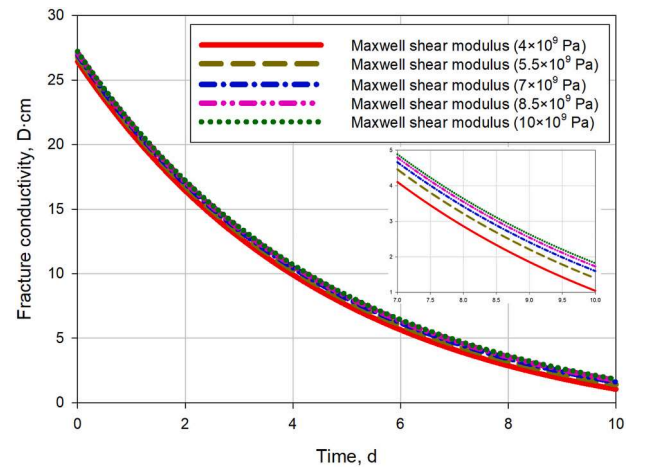


Fig. 13. Impacts of the Maxwell shear modulus on fracture conductivity.

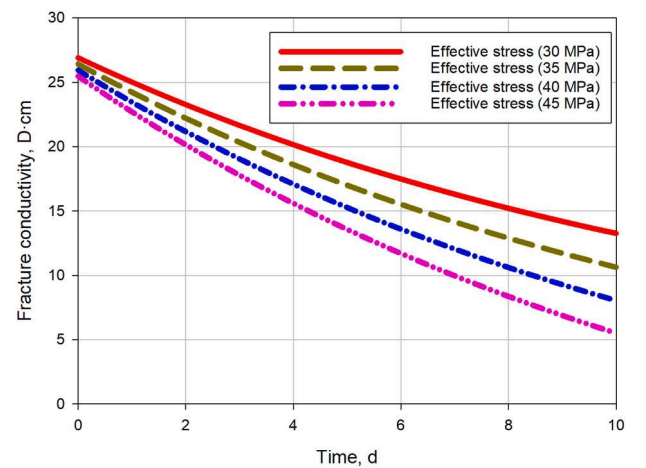


Fig. 14. Impacts of the effective stress on fracture conductivity.

slightly enhances the conductivity.

4.2. Impacts of effective stress

The stress state is a key factor dominating rock and proppant deformation and fracture conductivity. Here, the effective stress is the

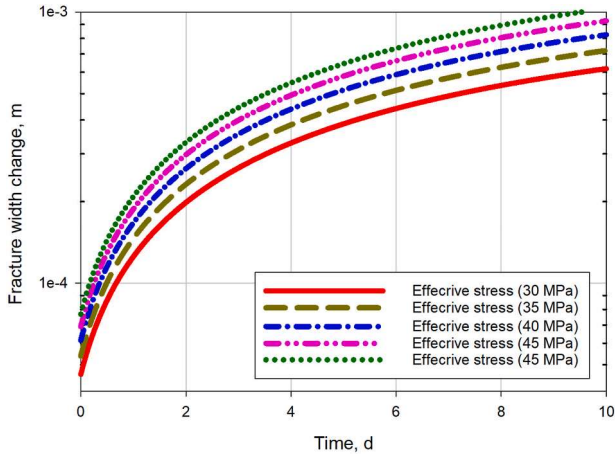


Fig. 15. Impacts of the effective stress on creep-induced fracture width variation (one-side).

Terzaghi effective stress which is difference between confining pressure fracture fluid pressure. With the effective stress increasing, fracture compaction is more significant, leading to lower conductivity, as shown in Fig. 14. As time increases, the conductivity drop becomes more noticeable. The change of effective stress has a marginal influence on the overall shape of the curves. When the effective stress is 30 MPa, the conductivity drop during the 10 days is 50.7 %. When the effective increases to 45 MPa, the conductivity drop reaches 78.4 % after 10-day compaction. Fig. 15 demonstrates the creep-induced fracture width variation under different effective stress conditions. As expected, a higher effective stress moves up the fracture width change curves and induces stronger creep deformation. By combining time, effective stress, and the corresponding conductivity, a three-dimensional (3D) conductivity diagram can be drawn, as illustrated in Fig. 16(a). Fig. 16(b) demonstrated the regions controlled by different mechanisms, such as effective stress and creep deformation.

4.3. Impacts of proppant size

Proppant size directly determines the initial aperture of the propped fracture segment. Here, the range of proppant radius is between 0.35 mm and 0.5 mm. which is reasonable for the 20/40 mesh proppants. As shown in Fig. 17, a larger proppant size provides higher overall fracture conductivity. With the increase of proppant size, the fracture conductivity enhancement becomes more significant. By comparing the conductivity reduction ratio, one can find that a larger proppant size offers higher residual fracture conductivity after 10 days (0.35 mm radius: 4.1 % residual conductivity, 0.40 mm radius: 16.6 % residual conductivity,

0.45 mm radius: 26.2 % residual conductivity, and 0.5 mm radius: 33.8 % residual conductivity). This indicates that larger-size proppants can better maintain fracture conductivity. However, the proppant size should match with the reservoir properties to avoid proppant plugging during fracturing operations.

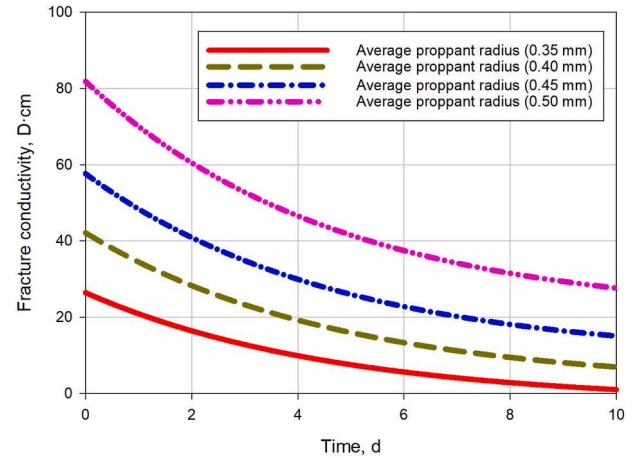


Fig. 17. Impacts of the proppant size on fracture conductivity.

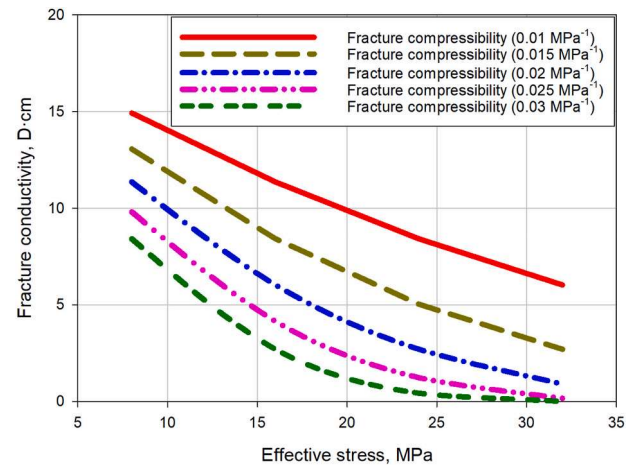


Fig. 18. Impacts of fracture compressibility on unpropped fracture conductivity.

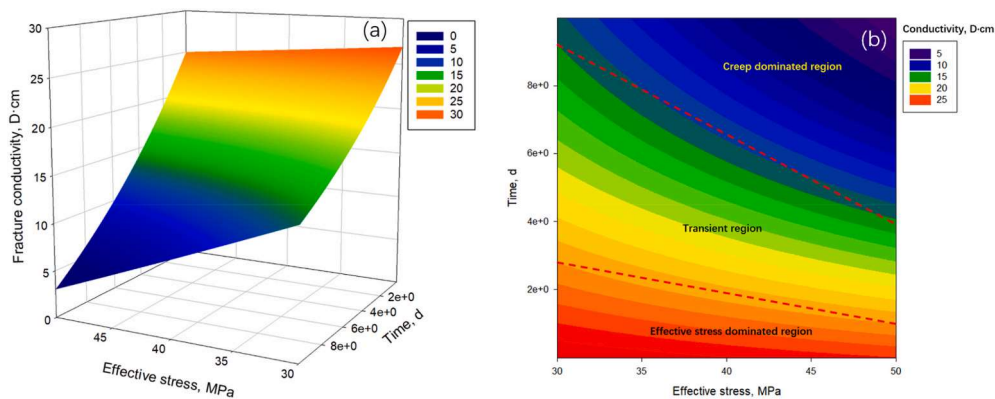


Fig. 16. Conductivity diagrams under different effective stress conditions: (a) 3D diagram and (b) 2D diagram.

4.4. Impacts of unpropped fracture compressibility

During the hydraulic fracturing operations, it is almost impossible to fully support the whole fracture network domain using proppants. Thus, a certain proportion of the fracture network is unpropped, leading to unpropped fracture segments within the fracture network. The compressibility of unpropped fractures is a dominant parameter for describing fracture aperture variation. Therefore, under a certain stress condition, unpropped fracture conductivity depends on fracture compressibility. The initial fracture aperture is set as 0.132 mm. As shown in Fig. 18, higher compressibility results in lower overall unpropped fracture conductivity. When the effective stress changes from 8 MPa to 32 MPa, the conductivity drops substantially. With the compressibility increasing from 0.01 MPa^{-1} to 0.03 MPa^{-1} , the conductivity decline rate rises from 59.6 % to 99.9 %. This indicates that the unpropped fracture conductivity tends to experience a complete loss when both the compressibility and the effective stress increment are large.

4.5. Fracture network conductivity

The fracture network consists of different types of propped and unpropped fracture segments. The overall conductivity is calculated through the hydraulic–electric analogies (Eqs. (29)–(33)). Here, the fracture network equivalent flow resistance shown in Fig. 7 is utilized to calculate fracture network conductivity. For each type of fracture segment, the length is set as 30 m ($l_2 = l_3 = l_4 = l_5 = 30 \text{ m}$, $l_1 = 20 \text{ m}$, and $l_1 = 10 \text{ m}$). The total fracture network length (l_t) is treated as the main fracture length (90 m). Fig. 19 compares the conductivity of the fracture network, different fracture segments, the main fracture, and the fracture branches. The overall fracture network conductivity is higher than the main fracture conductivity, fracture branch conductivity, and the conductivity of fracture segments 2 to 4. Only the conductivity of fracture segment 1 is larger than fracture network conductivity.

Fig. 20 demonstrates the conductivity ratios of the fracture network and different propped fracture segments. The conductivity ratio is the ratio of initial conductivity to the actual conductivity. It can be seen that using larger-size proppants can maintain the conductivity. The order of the degree of conductivity drop is: fracture segment 1 < fracture segment 2 < fracture segments 2 and 3. The overall fracture network conductivity reduction ratio is close to that of fracture segment 1, indicating that the main fracture segment with largest-size proppants dominant the conductivity maintenance ability.

Because this model is capable of characterizing time-dependent fracture conductivity, a conductivity map was drawn in Fig. 21. For

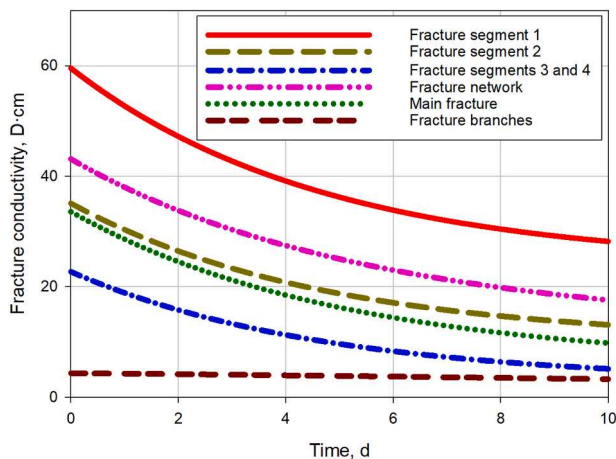


Fig. 19. Comparison of the conductivity of the fracture network and different fracture segments.

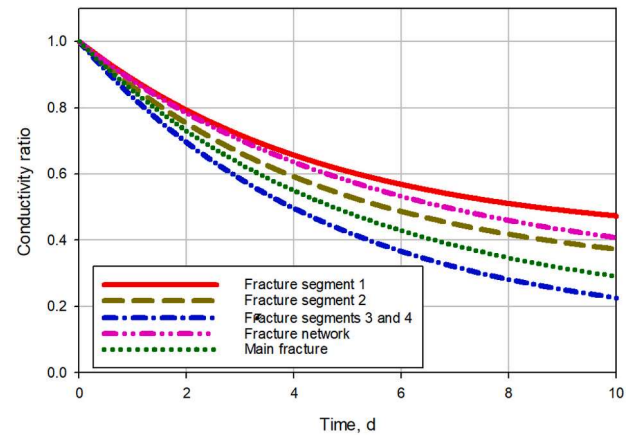


Fig. 20. Comparison of the conductivity ratios of fracture network and different fracture segments.

each stress level, all possible conductivity data points within a certain time can be plotted in the map. The vertical bars in Fig. 21 represent the 10-day conductivity variation data. For the fracture network, the range of conductivity variation extends as the effective stress increases. Fracture network conductivity map's upper and lower boundaries drop with effective increasing. A similar trend can be observed in fracture branch's conductivity map. Because the main fracture here is fully propped, the initial conductivity drop at each effective stress level is relatively small. Therefore, the upper boundary of the main fracture conductivity map is nearly a horizontal curve, while the lower boundary monotonically declines with increasing effective stress. The above results emphasize the significance of high-efficiency and full-domain proppant placement. The fracture conductivity map overlaps with the main fracture conductivity map as the effective stress is larger than 16 MPa.

5. Conclusions

This study introduces a general conductivity model that captures the time- and stress-dependent fracture network conductivity evolution caused by width and permeability variation of mixed-sized proppant-filled fracture segments and unpropped segments. The propped fracture segment conductivity model is verified against time-dependent fracture conductivity evolution data, while the unpropped fracture conductivity model has been verified in the literature by many scholars. Based on the above analysis, the following key conclusions can be drawn.

- (1) Results from the propped fracture segment conductivity model match well with the conductivity evolution data collected from two different shale samples. The conductivity reduction rates reach 86 % and 46 % within the first 2 days due to creep deformation. Proppant compaction exhibits the smallest influence on conductivity, while creep provides the heaviest impact on conductivity.
- (2) The increase of creep-deformation-related rock mechanical properties, such as the Maxwell viscosity and the Kelvin shear modulus, slows down the creep-induced conductivity decline. When the Kelvin shear modulus reaches $5.5 \times 10^8 \text{ Pa}$, the variation of the Maxwell shear modulus has a marginal on fracture conductivity.
- (3) The effective stress state controls the range of the conductivity change. The conductivity differences under different stress conditions gradually increase with time. The 10-day conductivity drop is 50.7 % under a 30 MPa effective stress condition, while this value increases to 78.4 % under a 45 MPa effective stress condition. The 3D conductivity diagram is proposed to illustrate the

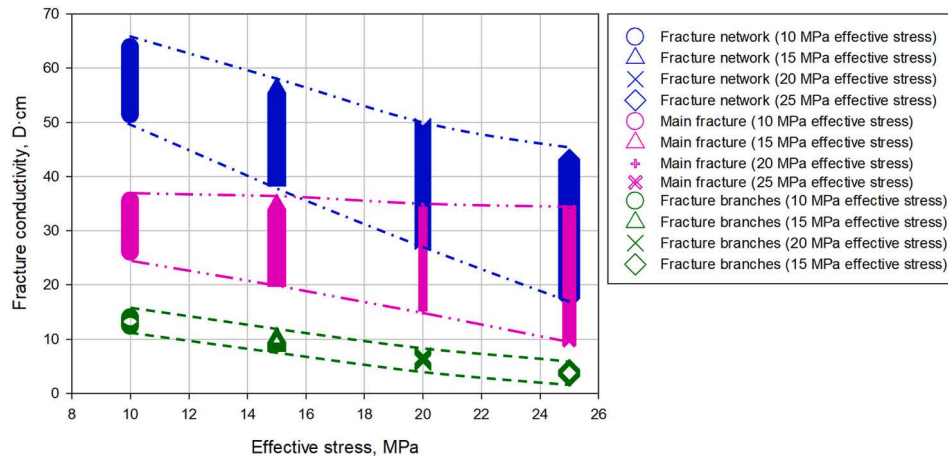


Fig. 21. Conductivity maps of the fracture network, main fracture, and fracture branches.

time-dependent conductivity evolution path under different effective stress conditions

- (4) With the same number of proppant layers, larger-size proppants offer higher overall conductivity and better main the fracture conductivity. When the proppant radius is 0.5 mm, the residual conductivity after 10 days is 33.8 % of the initial one. The 10-day residual conductivity drops to 4.1 % of the initial one when the proppant radius reduces to 0.35 mm. The model can help to select appropriate proppants according to specific reservoir properties.
- (5) Larger unpropped fracture compressibility reduces the overall unpropped fracture conductivity under identical stress conditions. When the compressibility increases to 0.03 MPa^{-1} , the conductivity loss reaches 99.9 % as the effective stress changes from 8 MPa to 32 MPa.
- (6) The 2D fracture conductivity map is proposed to demonstrate the range of conductivity evolution under different effective stress conditions in a certain period. Under low effective stress conditions, the order of conductivity is: fracture network > main fracture > fracture branch. However, when the effective stress is larger than 16 MPa, the fracture conductivity map overlaps with the main fracture conductivity map.

CRediT authorship contribution statement

Yingyan Li: Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis. **Chenlin Hu:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Jie Zeng:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Wenfeng Wang:** Writing – original draft, Validation, Investigation, Formal analysis. **Shiqian Xu:** Writing – original draft, Methodology, Investigation. **Yingfang Zhou:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization. **Fanhua Zeng:** Writing – review & editing, Writing – original draft, Methodology. **Jingpeng Wang:** Writing – review & editing, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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